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## חזקות ושורשים - פתרונות

פישוט חזקות עם מעריך שלילי:

- $4^{-3} = \left(\frac{1}{4}\right)^3 = \frac{1^3}{4^3} = \frac{1}{64}$
- $\left(\frac{1}{5}\right)^{-3} = 5^3 = 125$
- $\left(\frac{1}{5^{-3}}\right) = \frac{1}{\left(\frac{1}{5}\right)^3} = \frac{1}{\frac{1^3}{5^3}} = \frac{1}{\frac{1}{5^3}} = \frac{1}{1} = 125$
- $\left(\frac{2}{5}\right)^2 = \frac{2^2}{5^2} = \frac{4}{25}$
- $\left(\frac{1}{4}\right)^4 = \frac{1^4}{4^4} = \frac{1}{256}$
- $12^3 = 12 \times 12 \times 12 = 144 \times 12 = 1728$
- $\left(\frac{1}{2^{-4}}\right)^2 = \frac{1^2}{(2^{-4})^2} = \frac{1}{2^{-8}} = 2^8 = 256$
- $\left(\frac{1}{5^{-3}}\right)^{-2} = (5^3)^{-2} = 5^{-6} = \frac{1}{5^6}$
- $\left(\frac{2}{a}\right)^{-4} = \left(\frac{a}{2}\right)^4 = \frac{a^4}{2^4} = \frac{a^4}{16}$
- $\left(\frac{1}{x}\right)^{-1} = \frac{x^1}{1^1} = x$

כפל וחילוק חזקות:

- $3^4 \cdot 3^2 = 3^{2+4} = 3^6$
- $2^4 \cdot 2^3 = 2^{3+4} = 2^7$
- $5^2 \cdot 5^3 \cdot 5^4 = 5^{2+3+4} = 5^9$
- $x^4 \cdot x^1 = x^5$
- $3^x \cdot 3^7 = 3^{x+7}$
- $x^a \cdot x^b \cdot x^c = x^{a+b+c}$

$$7. \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^4 = \left(\frac{1}{2}\right)^{2+4} = \left(\frac{1}{2}\right)^6$$

$$8. \frac{3^4}{3^2} = 3^{4-2} = 3^2$$

$$9. \frac{2^2 \cdot 2^4}{2^3} = \frac{2^6}{2^3} = 2^3 = 8$$

$$10. \frac{3^4}{3^{-2}} = 3^6$$

$$11. \frac{x^{14}}{x^{12}} = x^{14-12} = x^2$$

$$12. \frac{x^{-2} \cdot x^{-5}}{x^{-3}} = \frac{x^{(-2)+(-5)}}{x^{-3}} = \frac{x^{-7}}{x^{-3}} = x^{-7-(-3)} = x^{-7+3} = x^{-4} = \left(\frac{1}{x}\right)^4 = \frac{1^4}{x^4} = \frac{1}{x^4}$$

$$13. \frac{x^2 \cdot 2^3}{x \cdot 2^2} = x^{2-1} \cdot 2^{3-2} = x^1 \cdot 2^1 = 2x$$

$$14. \frac{a^{-2} \cdot b^3 \cdot c^{2x}}{b^{-2} \cdot a^3 \cdot c^x} = a^{-2-3} \cdot b^{3-(-2)} \cdot c^{2x-x} = a^{-5} \cdot b^{3+2} \cdot c^x = \left(\frac{1}{a}\right)^5 \cdot b^5 \cdot c^x = \frac{1^5}{a^5} \cdot b^5 \cdot c^x = \frac{b^5 \cdot c^x}{a^5}$$

### חזקה על חזקה:

$$1. (3^4)^2 = 3^{4 \times 2} = 3^8$$

$$2. (4^5)^{-6} = 4^{-30}$$

$$3. (x^3)^c = x^{3 \times c} = x^{3c}$$

$$4. (2x^2)^x = (2 \cdot x^2)^x = 2^x \cdot x^{2 \cdot x}$$

$$5. (ab^b)^7 = (a \cdot b^b)^7 = a^7 \cdot b^{b \cdot 7} = a^7 b^{7b}$$

$$6. \left(\frac{(3^4)^2 \cdot (4^2)^2}{3^4 \cdot 4^2}\right)^{-2} = \left(\frac{3^{4 \cdot 2} \cdot 4^{2 \cdot 2}}{3^4 \cdot 4^2}\right)^{-2} = \left(\frac{3^8 \cdot 4^4}{3^4 \cdot 4^2}\right)^{-2} = (3^{8-4} \cdot 4^{4-2})^{-2} = (3^4 \cdot 4^2)^{-2} = 3^{-8} \cdot 4^{-4} = \frac{1}{3^8 \cdot 4^4}$$

ניתן גם להגיע לאותו הפתרון בדרך נוספת, על ידי פתיחת הסוגריים תחילה:

$$\left(\frac{(3^4)^2 \cdot (4^2)^2}{3^4 \cdot 4^2}\right)^{-2} = \left(\frac{3^{4 \cdot 2} \cdot 4^{2 \cdot 2}}{3^4 \cdot 4^2}\right)^{-2} = \left(\frac{3^8 \cdot 4^4}{3^4 \cdot 4^2}\right)^{-2} = \left(\frac{3^{8 \cdot (-2)} \cdot 4^{4 \cdot (-2)}}{3^{4 \cdot (-2)} \cdot 4^{2 \cdot (-2)}}\right) = \left(\frac{3^{-16} \cdot 4^{-8}}{3^{-8} \cdot 4^{-4}}\right) = 3^{-8} \cdot 4^{-4} = \frac{1}{3^8 \cdot 4^4}$$

$$7. (x^{-4})^6 = x^{-4 \cdot 6} = x^{-24} = \frac{1}{x^{24}}$$

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$$8. \left( \frac{(x^{-4})^2 \cdot y^2}{y^{-4} \cdot x^2} \right)^2 = \left( \frac{x^{-4 \cdot 2} \cdot y^2}{y^{-4} \cdot x^2} \right)^2 = \left( \frac{x^{-8} \cdot y^2}{y^{-4} \cdot x^2} \right)^2 = \frac{x^{-8 \cdot 2} \cdot y^{2 \cdot 2}}{y^{-4 \cdot 2} \cdot x^{2 \cdot 2}} = \frac{x^{-16} \cdot y^4}{y^{-8} \cdot x^4} = x^{-16-4} \cdot y^{4-(-8)} = x^{-20} \cdot y^{12} = \frac{y^{12}}{x^{20}}$$

דרך נוספת- נפטר קודם מקו השבר ואז נפתח את הסוגריים

$$\left( \frac{(x^{-4})^2 \cdot y^2}{y^{-4} \cdot x^2} \right)^2 = \left( \frac{x^{-4 \cdot 2} \cdot y^2}{y^{-4} \cdot x^2} \right)^2 = \left( \frac{x^{-8} \cdot y^2}{y^{-4} \cdot x^2} \right)^2 = (x^{-8 \cdot 2} \cdot y^{2 \cdot (-4)})^2 = (x^{-10} \cdot y^{2+4})^2 = x^{-10 \cdot 2} \cdot y^{6 \cdot 2} = \frac{y^{12}}{x^{20}}$$

$$9. (2(a-b)^2)^2 = 2^2 \cdot (a^2 - 2ab + b^2)^2 = 4 \cdot (a^2 - 2ab + b^2)^2$$

$$10. \left( \frac{a^{-2}b^3}{b^{-2}a^3} \right)^4 = \left( \frac{a^{-2 \cdot 4} \cdot b^{3 \cdot 4}}{b^{-2 \cdot 4} \cdot a^{3 \cdot 4}} \right) = \left( \frac{a^{-8} \cdot b^{12}}{b^{-8} \cdot a^{12}} \right) = a^{-20} \cdot b^{20}$$

**סידור חזקות לפי ערכן:**

$$1. \left( \left( \frac{1}{2} \right)^2 \right)^2 = \left( \frac{1}{2} \right)^{2 \cdot 2} = \left( \frac{1}{2} \right)^4 = \left( \frac{1^4}{2^4} \right) = \frac{1}{16}$$

$$2. \frac{3^2 \cdot 2^5}{(3^3)^{-\frac{1}{3}} \cdot \left( \frac{1}{2} \right)^{-5}} = \frac{3^2 \cdot 2^5}{3^{3 \cdot (-\frac{1}{3})} \cdot 2^5} = \frac{3^2 \cdot 2^5}{3^{-1} \cdot 2^5} = 3^{2-(-1)} \cdot 2^{5-5} = 3^{2+1} \cdot 2^0 = 3^3 \cdot 1 = 27$$

$$3. \frac{x^2 \cdot y}{\left( \frac{y}{x} \right)^{-2} \cdot y^3} - 1 = \frac{x^2 \cdot y}{\left( \frac{x}{y} \right)^2 \cdot y^3} - 1 = \frac{x^2 \cdot y}{\frac{x^2}{y^2} \cdot y^3} - 1 = \frac{x^2 \cdot y}{\frac{x^2 \cdot y^{\cancel{3}}}{y^{\cancel{2}}}} - 1 = \frac{x^2 \cdot y}{x^2 \cdot y} - 1 = 1 - 1 = 0$$

$$4. \frac{8^2 \cdot 1^5}{(2^3)^2 \cdot 2^0 \cdot 2^{-3}} = \frac{(2^3)^2 \cdot 1}{(2)^{2 \cdot 3} \cdot 1 \cdot 2^{-3}} = \frac{2^{3 \cdot 2} \cdot 1}{2^6 \cdot 2^{-3}} = \frac{2^6}{2^{6+(-3)}} = \frac{2^6}{2^3} = 2^{6-3} = 2^3 = 8$$

1. 3 < 2. 1 < 3. 4 < 4. 2